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## A Method Of Stating The Differential Form Of Gauss' Theorem

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**Abstract.** *The electric field created by a charge can be estimated and calculated using Coulomb's law, and if there are many charges, the electric field can be estimated and calculated using the principle of superposition. Therefore, many methods have been used to calculate the above problem, but the problem becomes complicated when calculated by these methods. Simpler than these methods is the theorem proposed by Gauss. With the help of this theorem, it is possible to calculate the field strength and the current of induction vectors of a charge located in a closed circuit. This gives Gauss's theorem in integral form in Maxwell's differential form.*

**Keywords:** *Gauss theorem, charged particle, electric field, principle of superposition of fields, current density, magnetic induction current, electric constant, divergence, charge density*

### Introduction

If the theorem proposed by the German physicist Gauss is used rather than the principle of superposition, which is used to calculate the field of electric charges in a vacuum, the calculation of the flow of the electrostatic field passing through any arbitrary closed surface is much simpler.

The flow of the electrostatic field intensity vector, which is released from the charge  $q$  located in the center of the closed spherical surface with radius  $r$  and is leaking from the closed spherical surface, is calculated as follows.

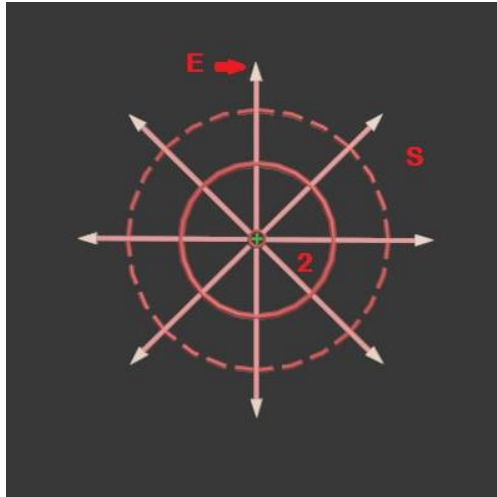
$$\Phi_E = \oint_S \vec{E}_n \, ds = \oint_S \vec{E} \, ds \quad (1)$$

In it, the flow of the electric field intensity vector passing through the closed surface  $S$  is determined as follows.

$$\overrightarrow{\Phi_E} = \oint_S \overrightarrow{E_n} ds = \oint_S \vec{E} ds = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0} \quad (2)$$

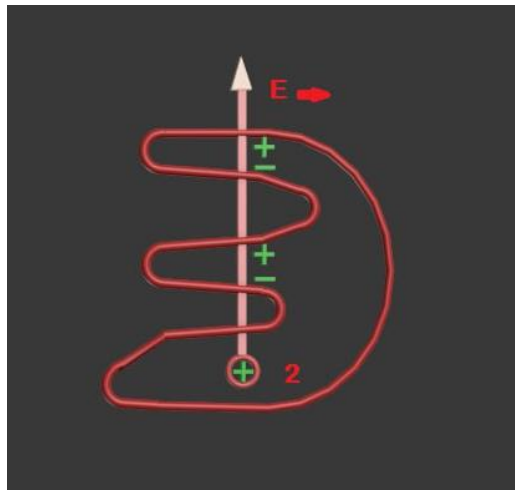
$4\pi r^2$  in this is the complete surface of a spherical surface of radius  $r$ .

This equation is valid for a closed surface of arbitrary shape. In fact, even when the spherical surface of Fig. 1 is surrounded by an arbitrary closed surface  $S$ , each field line vector emanating from the charge will leak through the arbitrary closed surface as well as through the spherical surface [1].



1-pacm

If the closed surface is of an arbitrary shape, and there is a charge  $q$  inside it, then the line of the field strength vector is directed in the position leaving the surface, entering the surface and leaving the surface again (Fig. 2) [3-5].



2-pacm

If there is no charge inside the closed surface, then the current of the field strength vector is zero:

$$\overrightarrow{\Phi_E} = \oint_S \vec{E} ds = 0 \quad (3)$$

Thus, if a charge  $q$  is located inside an arbitrary closed surface, then the current of the field strength vector is equal to  $\frac{q}{\epsilon_0}$ .

$$\overrightarrow{\Phi_E} = \oint_S \overrightarrow{E_n} ds = \oint_S \vec{E} ds = \frac{q}{\epsilon_0} \quad (4)$$

Метод

If  $n$  charges are located on a closed surface of an arbitrary shape, then, based on the principle of superposition, the total field intensity vector created by all charges is equal to the sum of the intensity vectors created by the charges:

$$\vec{E} = \sum_{i=1}^n \vec{E}_i \quad (5)$$

Therefore

$$\overrightarrow{\Phi_E} = \oint_S \vec{E} ds = \oint_S (\sum_{i=1}^n \vec{E}_i) dS = \sum_{i=1}^n \vec{E}_i dS \quad (6)$$

Then, based on the formula (4), each integral under the sum is equal to  $\frac{q}{\epsilon_0}$ . So

$$\overrightarrow{\Phi_E} = \oint_S \overrightarrow{E_n} ds = \oint_S \vec{E} ds = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i \quad (7)$$

This formula expresses Gauss's theorem for the electrostatic field in a vacuum:

the current of the electrostatic field intensity vector passing through an arbitrary surface in a vacuum is equal to the sum of the charges inside this surface divided by  $\epsilon_0$ .

As we know, in general, electric charges can be "swept" into the volume with some density, i.e.

$$\rho = \frac{dq}{dV} \quad (8)$$

This size can be different in different states of space. The sum of charges of volume  $V$  surrounded by a closed surface  $S$  is as follows.

$$\sum_{i=1}^n q_i = \int_V \rho dV \quad (9)$$

Using (8), Gauss's theorem can be written as follows.

$$\oint_S \vec{E} ds = \oint_S \overrightarrow{E_n} ds = \frac{1}{\epsilon_0} \int_V \rho dV \quad (10)$$

Finally, using the equation  $\vec{D} = \epsilon_0 \vec{E}$ , the last formula (9) is written as follows

$$\oint_S \vec{D} dS = \oint_V \rho dV \quad (11)$$

This equation is one of the integral forms of Gauss's theorem. The right-hand side of this equation is the volume integral of the charge density, and the left-hand side contains the flux of the electrostatic field induction vector over the closed surface. To bring these equations to the same integral, we use the following Ostrogradsky-Gaussian equation given in the appendix of [2].

$$\oint_S \vec{A} dS = \oint_V \text{div } \vec{A} dV \quad (12)$$

The meaning of this equation can be interpreted as follows:

The flow of the vector  $\vec{A}$  on the closed surface is equal to the flow of the vector  $\vec{A}$  on the volume, i.e. the amount of charge and charge density in the volume and the number of vectors emanating from them is the same amount that flows through the closed surface [6-10].

From above,  $\text{div } \vec{A}$  on the right-hand side of equation (11) represents the flow of vector  $\vec{A}$  through space:

$$\text{div } \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (13)$$

If we replace  $\vec{A}$  in equation (11) with the electric field induction vector,

$$\oint_S \vec{D} dS = \oint_V \text{div } \vec{D} dV \quad (14)$$

will be, using this we write (10) as follows:

$$\oint_V \text{div } \vec{D} dV = \oint_V \rho dV \quad (15)$$

now these equations can be added, since both are integral in volume.

In it

$$\oint_V \text{div } \vec{D} dV - \oint_V \rho dV = 0 \quad (16)$$

or

$$\oint_V (\text{div } \vec{D} - \rho) dV = 0 \quad (17)$$

this equation is equal to zero at this time if the expression under the integral sign is equal to zero, i.e.

$$\text{div } \vec{D} - \rho = 0$$

$$\text{div } \vec{D} = \rho \quad (18)$$

This equation is one of Maxwell's equations for electrostatics and is called the differential form of Gauss's theorem [11-12].

Conclusion

Therefore, wherever there is a charge density, a current of the electric field induction vector is generated around it. Given that this current is numerically equal to the charge density, the above equation can be written as follows.

$$\text{div } \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho \quad (19)$$

The meaning of this Maxwell's equation is that if the source of the electric field induction vector is a charge, the divergence is positive if the vectors radiate from the positive source, and negative if they converge on the charge.

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